

Common Fixed Point Theorem on Weak Reciprocally Continuous Mappings

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ABSTRACT: The purpose of this paper is to prove a unique common fixed point theorem in a fuzzy metric space using the concepts of weaker form of weak reciprocally continuous and weakly compatible mappings.

KEYWORDS: Fixed point, self maps, reciprocally continuous S-weak reciprocally continuous mappings, weakly compatible mappings.

AMS (2000) Mathematics Classification: 54H25, 47H10

I.INTRODUCTION

The notion of the fuzzy metric spaces was initiated by Zadeh [12], in the recent past many authors developed the theory of fuzzy sets and applications. Grabic [5] obtained the Banach contraction principle in fuzzy version. Kramosil and mechalek introduced the concept of fuzzy metric space. Mishra et al [8] initiated the concept of compatible mappings which was finally generalized by Bijendra Singh and Jain [2] by introducing the notion of weakly compatible mappings in Fuzzy metric space. Pant [9] obtained the concept of reciprocally continuous and proves fixed point theorem. In this paper we prove a unique fixed point theorem using weak reciprocally continuous with S-compatible (or A-compatible) and weakly compatible mappings.

II. DEFINITIONS AND PRELIMINARIES

Definition 2.1: A binary operation $*$: $[0,1] \times [0,1] \rightarrow [0,1]$ is called continuous t-norm if $*$ satisfies the following conditions:

- (i) $*$ is commutative and associative
- (ii) $*$ is continuous
- (iii) $a * 1 = a$ for all $a \in [0,1]$
- (iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$

Definition 2.2: A 3-tuple $(X, M, *)$ is said to be Fuzzy metric space if X is an arbitrary set, $*$ is continuous t- norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X, s, t > 0$

- (FM-1) $M(x, y, 0) = 0$
- (FM-2) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$
- (FM-3) $M(x, y, t) = M(y, x, t)$
- (FM-4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t+s)$
- (FM-5) $M(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$ is left continuous
- (FM-6) $\lim_{t \rightarrow \infty} M(x, y, t) \rightarrow 1$ as $t \rightarrow \infty$

International Journal of Innovative Research in Science, Engineering and Technology

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Website: www.ijirset.com

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The function $M(x, y, t)$ denote the degree of nearness between x and y with respect to 't'. We observe $M(x, y, t) = 1$ when $x = y$ for all $t > 0$.

Example 2.3: (Induced fuzzy metric space): Let (X, d) be a metric space defined $a * b = \min\{a, b\}$ for all $x, y \in X$ and $t > 0$,

$$M(x, y, t) = \frac{t}{t + d(x, y)} \quad \text{--- (a)}$$

Then $(X, M, *)$ is a fuzzy metric space. We call this fuzzy metric M induced by metric d the standard fuzzy metric. From the above example every metric induces a fuzzy metric but there exist no metric on X satisfying (a).

Definition 2.4: Let $(X, M, *)$ be a fuzzy metric space then a sequence $\langle x_n \rangle$ in X is said to be convergent to a point $x \in X$, if for each $\epsilon > 0, t > 0$ there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x, t) > 1 - \epsilon$ for all $n \geq n_0$

Definition 2.5: Let $(X, M, *)$ be a fuzzy metric space then a sequence $\langle x_n \rangle$ in X is said to be Cauchy sequence, if for each $\epsilon > 0, t > 0$ there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for all $n \geq n_0$

Definition 2.6: A fuzzy metric space $(X, M, *)$ is said to be complete if every Cauchy sequence is convergent to a point in X .

Lemma 2.7: For all $x, y \in X, M(x, y, \cdot)$ is non decreasing.

Lemma 2.8 let $(X, M, *)$ be a fuzzy metric space if there exists $k \in (0, 1)$ such that $M(x, y, kt) \geq M(x, y, t)$ then $x = y$.

Proposition 2.9 : In the fuzzy metric space $(X, M, *)$ if $a * a \geq a$ for all $a \in [0, 1]$ then $a * b = \min\{a, b\}$

Definition 2.10: Two self maps S and T of a fuzzy metric space $(X, M, *)$ are said to be compatible mappings if $\lim_{n \rightarrow \infty} M(STx_n, TSx_n, t) = 1$, whenever $\langle x_n \rangle$ is a sequence in X such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = z$ for some $z \in X$.

Definition 2.11: Two self maps S and T of a fuzzy metric space $(X, M, *)$ are said to be S-compatible mappings if $\lim_{n \rightarrow \infty} M(STx_n, TTx_n, t) = 1$, whenever $\langle x_n \rangle$ is a sequence in X such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = z$ for some $z \in X$.

Definition 2.12: Two self maps S and T of a fuzzy metric space $(X, M, *)$ are said to be Reciprocally continuous mappings if $\lim_{n \rightarrow \infty} M(STx_n, Sz, t) = 1$ and $\lim_{n \rightarrow \infty} M(TSx_n, Tz, t) = 1$, whenever $\langle x_n \rangle$ is a sequence in X such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = z$ for some $z \in X$.

Definition 2.13: Two self maps S and T of a fuzzy metric space $(X, M, *)$ are said to be S-weak reciprocally continuous mappings if $\lim_{n \rightarrow \infty} M(STx_n, Sz, t) = 1$, whenever $\langle x_n \rangle$ is a sequence in X such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = z$ for some $z \in X$.

Definition 2.14: Two self maps S and T of a fuzzy metric space $(X, M, *)$ are said to be weakly compatible if they commute at their coincidence point. i.e if $Su = Tu$ for some $u \in X$ then $STu = TSu$.

International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 6, Issue 8, August 2017

III. IN 2005 BIJENDRA SINGH ET AL [2] PROVED THE FOLLOWING THEOREM

3.1 Theorem(A): Let A, B, S and T are self maps of a complete Fuzzy metric space $(X, M, *)$ with continuous t-norm $*$ defined by $a*b = \min \{a, b\}$, where $a, b \in [0, 1]$, satisfying the conditions

(3.1.1) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$

(3.1.2) S and T are continuous

(3.1.3) the pairs (A, S) and (B, T) compatible pairs of maps

$$(3.1.4) [M(Ax, By, kt)] \geq \min \{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(By, Sx, 2t), M(Ax, Ty, t)\}$$

for all x, y in X, $k \in (0, 1), t > 0$

(3.1.5) for all x, y in X $\lim M(x, y, t) \rightarrow 1$ as $t \rightarrow \infty$

then A, B, S and T have a unique common fixed point z in X

Now the generalization of the Theorem 3.1 as follows

IV. MAIN RESULT

4.1 Theorem: Let A, B, S and T are self maps of a complete Fuzzy metric space $(X, M, *)$ with continuous t-norm $*$ defined by $a*b = \min \{a, b\}$ where $a, b \in [0, 1]$, satisfying the conditions

(4.1.1) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$

(4.1.2) In the pairs (A, S), S-weak reciprocally continuous and S-compatible (or) A-weak reciprocally continuous and A-compatible

(4.1.3) (B, T) weakly compatible

$$(4.1.4) [M(Ax, By, kt)] \geq \min \{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(By, Sx, 2t), M(Ax, Ty, t)\}$$

where for all x, y in X, $k \in (0, 1), t > 0$

(4.1.5) for all x, y in X $\lim M(x, y, t) \rightarrow 1$ as $t \rightarrow \infty$

then A, B, S and T have a unique common fixed point z in X

(4.1.6) Lemma: Let A, B, S and T be self mappings from a complete metric space (X, d) into itself satisfying the conditions (4.1.1) and (4.1.4) then the sequence $\{y_n\}$ defined by $y_{2n} = Ax_{2n} = Tx_{2n+1}$ and $y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$ for $n \geq 0$, relative to four self maps is a Cauchy sequence in X.

Proof:

Let x_0 be any arbitrary point of X from the condition (4.1.1) there exists $x_1, x_2 \in X$ such that $Ax_0 = Tx_1$ and $Bx_1 = Sx_2$. Inductively we construct a sequence $\langle x_n \rangle$ and $\langle y_n \rangle$ in X such that $y_{2n} = Ax_{2n} = Tx_{2n+1}$ and $y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$ for $n \geq 0$.

By taking $x = x_{2n}$ $y = x_{2n+1}$ in the inequality (4.1.4)

International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 6, Issue 8, August 2017

$$\begin{aligned}
 &M(y_{2n}y_{2n+1}, kt) \\
 &= [M(Ax_{2n}, Bx_{2n+1}, kt)] \\
 &\geq \min \{M(Sx_{2n}, Tx_{2n+1}, t), M(Ax_{2n}, Sx_{2n}, t), M(Bx_{2n+1}, Tx_{2n+1}, t), M(Bx_{2n+1}, Sx_{2n}, 2t), M(Ax_{2n}, Tx_{2n+1}, t)\} \\
 &\geq \min \{M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n-1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n+1}, y_{2n-1}, 2t), M(y_{2n}, y_{2n}, t)\} \\
 &\geq \min \{M(y_{2n-1}, y_{2n}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n+1}, y_{2n-1}, 2t), 1\} \\
 &\geq \min \{M(y_{2n-1}, y_{2n}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n-1}, y_{2n}, t), 1\} \\
 &\geq \min \{M(y_{2n-1}, y_{2n}, t), M(y_{2n+1}, y_{2n}, t), 1\} \\
 &\geq M(y_{2n-1}, y_{2n}, t) \\
 &\text{In general } M(y_n y_{n+1}, kt) \geq M(y_{n-1}, y_n, t) \quad \dots\dots\dots(1)
 \end{aligned}$$

To prove $\{y_n\}$ is Cauchy sequence we prove $M(y_n, y_{n+m}, t) > 1 - \epsilon$ (2) is true for all $n \geq n_0$ and for every $m \in \mathbb{N}$.

Here we use induction method

From(1) we have

$$[M(y_n, y_{n+1}, t)] \geq M(y_{n-1}, y_n, \frac{t}{k}) \geq M(y_{n-2}, y_{n-1}, \frac{t}{k^2}) \geq \dots \geq M(y_1, y_2, \frac{t}{k^n}) \text{ as } n \rightarrow \infty$$

i.e for $t > 0, \epsilon \in (0, 1)$ we can choose $n_0 \in \mathbb{N}$ such that $M(y_n, y_{n+1}, t) > 1 - \epsilon$

Thus (2) is true for $m=1$.

suppose (2) is true for m then we shell show that it is also true for $m+1$

using the definition of fuzzy metric space (1) and(2) we have

$$M(y_n, y_{n+m}, t) \geq \min \{M(y_n, y_{n+m}, t/2), M(y_{n+m}, y_{n+m+1}, t/2)\} > 1 - \epsilon$$

Hence (2) is true for $m+1$. Thus $\{y_n\}$ is a Cauchy sequence.

By completeness of $(X, M, *)$, $\{y_n\}$ converges to some point z in X

Therefore $Ax_{2n} \rightarrow z, Tx_{2n+1} \rightarrow z, Bx_{2n+1} \rightarrow z, Tx_{2n+1} \rightarrow z$ as $n \rightarrow \infty$.

Proof of Theorem 4.1:

From the Lemma we have $Ax_{2n} \rightarrow z, Tx_{2n+1} \rightarrow z, Bx_{2n+1} \rightarrow z, Tx_{2n+1} \rightarrow z$ as $n \rightarrow \infty$

Case (i): S-weak reciprocally continuous implies $\lim_{n \rightarrow \infty} SAx_n \rightarrow Sz$

Since S-compatibility $\lim_{n \rightarrow \infty} M(SAx_{2n}, AAx_{2n}, t) = 1$ implies $\lim_{n \rightarrow \infty} SAx_{2n} = \lim_{n \rightarrow \infty} AAx_{2n} = Sz$

To prove $Sz=z$

Put $x=Ax_{2n}, y= x_{2n+1}$ in the inequality (4.1.4)

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$$[M(AAx_{2n}, Bx_{2n+1}, kt)] \geq \min \left\{ \begin{array}{l} M(SAx_{2n}, Tx_{2n+1}, t), M(AAx_{2n}, SAx_{2n}, t), \\ M(Bx_{2n+1}, Tx_{2n+1}, t), M(Bx_{2n+1}, SAx_{2n}, 2t), M(AAx_{2n}, Tx_{2n+1}, t) \end{array} \right\}$$

$$[M(Sz, z, kt)] \geq \min \{M(Sz, z, t), M(Sz, Sz, t), M(z, z, t), M(z, Sz, 2t), M(Sz, z, t)\}$$

$$[M(Sz, z, kt)] \geq M(Sz, z, t)$$

$$Sz = z$$

To prove $Az=z$ put $x=z, y= x_{2n+1}$ in the inequality (4.1.4)

$$[M(Az, Bx_{2n+1}, kt)] \geq \min \left\{ \begin{array}{l} M(Sz, Tx_{2n+1}, t), M(Az, Sz, t), M(Bx_{2n+1}, Tx_{2n+1}, t), \\ M(Bx_{2n+1}, Sz, 2t), M(Az, Tx_{2n+1}, t) \end{array} \right\}$$

$$[M(Az, z, kt)] \geq \min \{M(z, z, t), M(Az, z, t), M(z, z, t), M(z, z, 2t), M(Az, z, t)\}$$

$$[M(Az, z, kt)] \geq M(Az, z, t)$$

$$Az = z$$

$$\therefore Az = Sz = z$$

From the condition $A(X) \subseteq T(X)$ implies there exists $u \in X$ such that $Tu=Az=z$

To prove $Bu=z$ put $x= z, y=u$ in the inequality (4.1.4)

$$[M(Az, Bu, kt)] \geq \min \{M(Sz, Tu, t), M(Az, Sz, t), M(Bu, Tu, t), M(Bu, Sz, 2t), M(Az, Tu, t)\}$$

$$[M(z, Bu, kt)] \geq \min \{M(z, z, t), M(z, z, t), M(Bu, z, t), M(Bu, z, 2t), M(z, z, t)\}$$

$$[M(z, Bu, kt)] \geq M(Bu, z, t)$$

$$Bu = z$$

Since the pair (B, T) is weakly compatible gives $BTu=TBu$ implies $Bz=Tz$.

To prove $Bz=z$

Put $x=z, y= z$ in the inequality (4.1.4)

$$[M(Az, Bz, kt)] \geq \min \{M(Sz, Tz, t), M(Az, Sz, t), M(Bz, Tz, t), M(Bz, Sz, 2t), M(Az, Tz, t)\}$$

$$[M(z, Bz, kt)] \geq \min \{M(z, Bz, t), M(z, z, t), M(Bz, Bz, t), M(Bz, z, 2t), M(z, Bz, t)\}$$

$$[M(z, Bz, kt)] \geq M(z, Bz, t)$$

$$Bz = z$$

$$\therefore Bz = Tz = z$$

Case (ii) : Since the pair (A, S) is A-weak reciprocally continuous which implies $\lim_{n \rightarrow \infty} ASx_n \rightarrow Az$

and since A-compatibility implies $\lim_{n \rightarrow \infty} M(ASx_{2n}, SSx_{2n}, t) = 1$

$$\text{therefore } \lim_{n \rightarrow \infty} ASx_{2n} = \lim_{n \rightarrow \infty} SSx_{2n} = Az$$

To prove $Az=z$ put $x=Sx_{2n}, y= x_{2n+1}$ in the inequality (4.1.4)

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$$[M(ASx_{2n}, Bx_{2n+1}, kt)] \geq \min \left\{ \begin{array}{l} M(SSx_{2n}, Tx_{2n+1}, t), M(ASx_{2n}, SSx_{2n}, t), \\ M(Bx_{2n+1}, Tx_{2n+1}, t), M(Bx_{2n+1}, SSx_{2n}, 2t), M(ASx_{2n}, Tx_{2n+1}, t) \end{array} \right\}$$

$$[M(Az, z, kt)] \geq \min \{M(Az, z, t), M(Az, Az, t), M(z, z, t), M(z, Az, 2t), M(Az, z, t)\}$$

$$[M(Az, z, kt)] \geq M(Az, z, t)$$

$$Az = z$$

Since the condition $A(X) \subseteq T(X)$ implies there exists $u \in X$ such that $Tu = Az = z$

To prove $Bu = z$

Put $x = x_{2n}, y = u$ in the inequality (4.1.4)

$$[M(Ax_{2n}, Bu, kt)] \geq \min \{M(Sx_{2n}, Tu, t), M(Ax_{2n}, Sx_{2n}, t), M(Bu, Tu, t), M(Bu, Sx_{2n}, 2t), M(Ax_{2n}, Tu, t)\}$$

$$[M(z, Bu, kt)] \geq \min \{M(z, z, t), M(z, z, t), M(Bu, z, t), M(Bu, z, 2t), M(z, z, t)\}$$

$$[M(z, Bu, kt)] \geq M(Bu, z, t)$$

$$Bu = z$$

Since the pair (B, T) is weakly compatible, $BTu = TBu$ gives $Bz = Tz$

To prove $Bu = z$

put $x = x_{2n}, y = u$ in the inequality (4.1.4)

$$[M(Ax_{2n}, Bz, kt)] \geq \min \{M(Sx_{2n}, Tz, t), M(Ax_{2n}, Sx_{2n}, t), M(Bz, Tz, t), M(Bz, Sx_{2n}, 2t), M(Ax_{2n}, Tz, t)\}$$

$$[M(z, Bz, kt)] \geq \min \{M(z, Bz, t), M(z, z, t), M(Bz, Bz, t), M(Bz, z, 2t), M(z, Bz, t)\}$$

$$[M(z, Bz, kt)] \geq M(Bz, z, t)$$

$$Bz = z$$

$$\therefore Bz = Tz = z$$

Since the condition $B(X) \subseteq S(X)$ implies there exists $v \in X$ such that $Sv = Bz = z$

The pair (A, S) is A-compatibility implies $\lim_{n \rightarrow \infty} M(ASx_{2n}, SSx_{2n}, t) = 1 \implies \lim_{n \rightarrow \infty} ASx_{2n} = \lim_{n \rightarrow \infty} SSx_{2n}$.

Let $v = \lim_{n \rightarrow \infty} x_{2n}$ then $ASv = SSv$ this implies $Az = Sz$

$$\therefore Az = Sz = z.$$

Since $Az = Bz = Sz = Tz = z$ implies z is a common fixed point of A, B, S and T .

Uniqueness:

Let w be a another common fixed point of above self maps then we have $Aw = Bw = Sw = Tw = w$

Put $x = z, y = w$ in the inequality (4.1.4)

$$[M(Az, Bw, kt)] \geq \min \{M(Sz, Tw, t), M(Az, Sw, t), M(Bw, Tw, t), M(Bw, Sz, 2t), M(Az, Tw, t)\}$$

$$[M(z, Bw, kt)] \geq M(z, Bw, t)$$

Hence $z = w$.

4.1.6 Example: Let $X = [-1, 1]$, $M(x, y, t) = \frac{t}{t + d(x, y)}$ where $d(x, y) = |x - y|$

International Journal of Innovative Research in Science, Engineering and Technology

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$$Ax=Bx=\begin{cases} \frac{1}{10} & \text{if } -1 < x < \frac{1}{6} \\ \frac{1}{6} & \text{if } \frac{1}{6} \leq x < 1 \end{cases} \quad Sx=Tx=\begin{cases} \frac{1}{8} & \text{if } -1 < x < \frac{1}{6} \\ \frac{1}{3}-x & \text{if } \frac{1}{6} \leq x < 1 \end{cases}$$

Then $A(X) = B(X) = \left\{ \frac{1}{10}, \frac{1}{6} \right\}$ while $S(X) = T(X) = \left\{ \frac{1}{8} \cup \left[-\frac{2}{3}, \frac{1}{6} \right] \right\}$ so that the conditions

$A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$ are satisfied .

take a sequence $x_n = \left(\frac{1}{6} + \frac{1}{n} \right)$ for $n \geq 1$, then $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = 1/6$ and $\lim_{n \rightarrow \infty} ASx_n = 1/8$ also $SAx_n = 1/6$,

$\lim_{n \rightarrow \infty} M(SAx_n, AAx_n, t) = 1$ implies S -compatible

$\lim_{n \rightarrow \infty} SAx_n \rightarrow S(1/6)$ implies S -weakly reciprocally continuous

but so that $\lim_{n \rightarrow \infty} M(ASx_n, SAx_n, t) \neq 1$.

In the pairs (A, S) , S -weakly reciprocally continuous and S -compatible (B, T) are weakly compatible as they commute at coincident point $1/6$.

Also note that none of the mappings are continuous. Hence $1/6$ is the unique common fixed point of A, B, S and T .

V. CONCLUSION

Unique fixed point exists for Theorem 4.1 where the none of the mappings are continuous. And also from the Example 4.1.6 we have observed that the pair of S -compatible Mappings is need not be compatible mappings.

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